DROPOUTS IN COMBAT: A STOCHASTIC MODEL

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THESIS

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ABSTRACT

Stochastic models of combat which have been proposed consider that the personnel on the battlefield participate fully in the battle. Observation of behavior in combat indicates that a significant portion of the forces involved do not fire their weapons. These personnel are termed dropouts and the drop-out phenomenon is discussed. A probabilistic model of combat which considers both casualty generation and the drop-out phenomenon is proposed. This combat model has time variable rates associated with the attrition of the opposing sides. With the relaxation of this assumption, the stochastic combat model is shown to have an expected value which approximates Lanchester's Linear Law.

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SUMMARY

Post-combat interviews in World War II and Korea establish that a significant portion of the infantrymen involved in combat fail to fire upon the enemy when it is possible to do so. These personnel, termed dropouts, have an appreciable effect on the attrition of forces in combat.

Assuming that the combat process is orderly and that instantaneous rates of attrition exist, the probability of remaining in some state for an interval of time may be developed as well as conditional transition probabilities. With the additional assumption of the Markov property, the stochastic process may be completely characterized.

When the instantaneous rates are constant, the process has expected value which approximates Lanchester's Linear Law. The drop-out phenomenon is shown to affect the efficiency (exchange ratio) parameter of Lanchester's Law.

I. INTRODUCTION

As long as conflict in human society is resolved by the continuing application of military force, there are many reasons a model of combat is desirable. In time of peace, it is not possible to observe the combat process. In wartime, it may be extremely costly to observe the effects of various changes upon the process. To avoid both the expense of maintaining an excessively large military establishment and the losses forced by insufficient strength, force level decision makers should have some procedure by which to predict the effects of contemplated actions. In order to prepare for future battles and to effectively utilize those resources allocated to combat, a method by which future outcomes of the process may be predicted is required by the tactician. Non-combat experiments, such as exercises to test battle procedures or laboratory examination of military components, must be related to the combat process to have military value. Also, by abstracting the essential characteristics from the process to form a model, the phenomena of combat may be better understood.

Early attempts at predicting the outcomes of battle included interpreting omens and consulting oracles by the ancients as well as applying rules of thumb which have their modern equivalent. For example, it is said that for conventional forces to win a guerrilla (skirmishing war by small forces of irregular soldiers behind the lines of an army) requires a numerical superiority of ten to one (Ref. 20), or to conduct a successful assault requires a superiority of two or three to one (Ref. 6).

In 1916, F. W. Lanchester (Ref. 15) formulated an analysis of the effects of firepower in combat. By assuming that the rate of attrition for a force engaged in combat depended upon the strength of the opposition, he obtained a system of differential equations describing the attrition of the two forces. Many theoreticians have combined and extended the deterministic models of Lanchester (Ref. 9).

During a revival of interest in Lanchester's theory of combat in the time of World War II, the strictly deterministic treatment proposed by Lanchester was modified by analysts when the inherent probabilistic properties of the combat process were recognized. A stochastic analysis of Lanchester's theory of combat was then evolved by the Operations Research Group in the Department of the Navy in which Lanchester's deterministic equations were considered to represent the expected value of the combat process (Ref. 7). Snow (Ref. 23) and Morse and Kimball (Ref. 18) demonstrated approximations to the Lanchester equations for the expected value of simple, stationary, Markov combat processes.

Some investigators have examined historical information to discover the properties of the combat process. Observation of the process is necessary to formulate the hypotheses concerning the interdependence of events upon which a model may be constructed.

To verify the relationships postulated by Lanchester, Engel examined information from the assault on Iwo Jima in World War II. Lanchester's Square Law was shown to fit the then available data for the battle (Ref. 11). R. J. Best analyzed later Iwo Jima data and found considerable variability in the fraction of battalions actively engaged. Of all the battalion days during the first twenty days, the fraction of battalions engaged ranged from 1/3 to 11/12 except for "day fourteen" which was a day of reorganization when no attacks occurred. The average battalion participation was fifty per cent. Best also found that replacement units ashore were used initially as stevedore labor and were withheld from the battle for some time. This analysis of the battle also showed that the intensity of the combat steadily diminished and that the Japanese weapon mix changed during the course of the battle (Ref. 4). The findings of Best suggest that the extremely good fit achieved by Engel was coincidental and that the model used did not portray the process as the action developed.

Another property observed by investigators is that few historical battles continue to the annihilation of one of the participants. Weiss examined casualty data from the Civil War and was able to formulate a stopping rule for the process. By assuming that the probability of termination, that is, loss, surrender, or disengagement, depends only upon fractional casualties sustained up to that moment, the data was representable by the function $\exp(-kf^3)$, where f is the casualty fraction and k is a constant associated with one of two battle categories, assaults on fortified lines or meeting engagements. Weiss also concluded from the U.S. Civil War data that for assaults on fortified lines, the losses of the assaulting force were proportional to the number of defenders. For other battles in that war, casualty ratios appeared to have no relationship with force ratios although the probability of winning was dependent upon force ratio (Ref. 26).

Weiss also examined the air action known as the Battle of Britain in World War II. The data shows that the number of aircraft lost was proportional to the number committed for each side. This result does not follow from either the Lanchester linear law where the exchange ratio is independent of force ratio, or the Lanchester square law where exchange ratio varies inversely as force ratio. Weiss suggests that this phenomenon of losses increasing with the number committed is the result of nonhomogeneous forces or the law of diminishing returns, that is, forces become less efficient as they grow larger (Ref. 27).

Best analyzed extensive data from World War II: American, German, British, and Russian infantry experience. In addition, he considered information about American casualties in Korea. He found that a wide-ranging quasi-exponential distribution is typical of the daily number of battle casualties sustained by individual units: companies, battalions, and divisions. He also found that for particular companies, casualty incidence declines with time in battle for both offense and defense (Ref. 4).

The success of these investigators in aggregating combat data and formulating conclusions from that data seems to indicate that a mathematical model of combat has some promise as a method for predicting future outcomes of the combat process.

The models of combat which have been proposed assume that all members of the opposing sides participate actively in the process. This thesis proposes a stochastic model which assumes that non-fighters are present on the battle field. Additionally, a special case of this stochastic model is shown to have expected value which approximates Lanchester's Linear Law. Observations of past battles have not provided sufficient numerical data so that the parameters of these models may be specified. For this reason, no predictions are made.

II. THE DROP-OUT PHENOMENON

Observations of the combat process indicate that a significant portion of the soldiers engaged are not full participants in the battle. These individuals are expected to use the weapons with which they are equipped and trained, and have an opportunity for the use of the weapon, yet never fire on the enemy. These we shall call dropouts. Although the combat process might end in any of the following conditions for the dropout, he is not classifiable as a deserter, prisoner, casualty, or psychiatric casualty. He is the non-fighter.

After conducting post-combat interviews with approximately 400 infantry companies in both Europe and the Central Pacific during World War II, Marshall concluded that on an average less than 15 per cent of the survivors of the battles had actually fired at the enemy during an engagement. The best performance observed was that 25 per cent of one company had made at least some use of a weapon. The actions considered were, for the most part, decisive local actions in which the company had been hard pressed and at least 80 per cent of the men involved had been in a position where it should have been possible to fire during the battle (Ref. 16).

The methodology used (interviews at a full assembly) makes these figures somewhat suspect, but any bias must favor a higher percentage of drop-outs as the men surely realized that participation in the fight was image enhancing. Also, if a man had participated with a weapon in any way he was included with the active fighters.

In a survey of 277 wounded combat veterans in the European Theater of Operations in August 1944, 65 per cent of the men admitted having had at least one experience in combat in which they were unable to perform adequately because of intense fear (Ref. 24). Although the sample is limited and adequate performance is not defined, this may be another indication of the drop-out phenomenon.

Marshall went to Korea in the winter of 1950-1951 and used the technique of post-combat interviews at full assembly to analyze infantry operations and weapons use. He concluded that between 12 and 20 per cent of the men showed some initiative and that an additional 25 to 35 per cent took some part in fire actions, for a total percentage of between 37 and 55 per cent actually using some weapon. In Korea, Marine and Army units were aware of the fire factor and "anxious to know how they stacked up against the average." During World War II, the officers and non-commissioned officers had a general impression that all men were participants while in Korea, having been alerted to the problem, they often moved and checked to assure that the men were using their weapons (Ref. 17).

Within the American Army during World War II, it was noted that the drop-out phenomenon was most prevalent among individual riflemen. Marshall advocated equipping the reluctant rifleman with the heavier weapons organic to the company (Ref. 16). In many Korean engagements grenadiers, mortars, machine guns, carbines, automatic rifles, bazookas and recoilless weapons exhausted available ammunition, yet no case was found where a company's rifle ammunition was completely depleted, however desperate the action (Ref. 17). An examination of Lanchester's square law leads to the conclusion that the increasing of manpower (concentration) gives a greater benefit than the increasing of force efficiency. If there is a causal relationship between the drop-out phenomenon and individual weapons efficiency, then this conclusion may not be true.

The United States Army tacitly confirmed the drop-out phenomenon in the Korean War by sending a research team to the battlefield to report on the characteristics of good and poor combat performance (Ref. 10). Another project of the Korean War Era, Fighter, sought a psychological testing method to discriminate between fighters and non-fighters prior to combat. This is another indication that the American Army was concerned with the drop-out phenomenon.

There is a widely held idea among military men that the use of firepower in the general area of enemy positions will serve to reduce enemy participation in the battle. This idea is put into action in the war in Viet Nam with the use of harrassing and interdicting missions for artillery, flack support for aircraft missions, and suppressive fire by infantry and supporting arms during assault. The idea that suppressive fire decreases enemy participation in the battle seems to support the existence of the drop-out phenomenon.

Shellard and Welch analyzed the effect of suppressive fire against German troops occupying open positions in three World War II battles. These battles were an assault on Geilenkirschen, an assault near Wyler, and the Normandy invasion. All attacks were against infantry positions and were characterized by different amounts of fire by the Allies on various areas under attack. The measure of the effect of the bombardment was considered to be the number of attacker casualties per defender. The data is summarized in Figure 1.

Accounts of the actions are illuminating and give insight into the drop-out phenomenon. At those areas which received very heavy suppressive fire at Geilenkirschen, the British infantry found "not the slightest resistence" and attacking troops described the defenders as "absolutely yellow colored." German casualties were estimated to be between ten and fifteen per cent, yet "prisoners interrogated later were clearly very shaken physically and said that they had felt quite overwhelmed with a sense of helplessness in the face of immense superiority." At other areas, German casualties were estimated at five per cent, yet defenders offered little resistance and expressed the same feeling of helplessness.

At Wyler, similar effects were noted although casualties caused by the neutralizing fire were only two or three per cent. When the suppressive fire was followed quickly by the infantry assault, the defenders surrendered readily. However, some of the attackers were slowed by anti-personnel mines and others lagged behind the bombardment. Those battalions which

G - Geilenkirschen attacks

N - Normandy attacks

W - Wyler attacks immediately following bombardment

X - Wyler attacks slowed by anti-personnel mines

L - Wyler attacks in which assault lagged bombardment

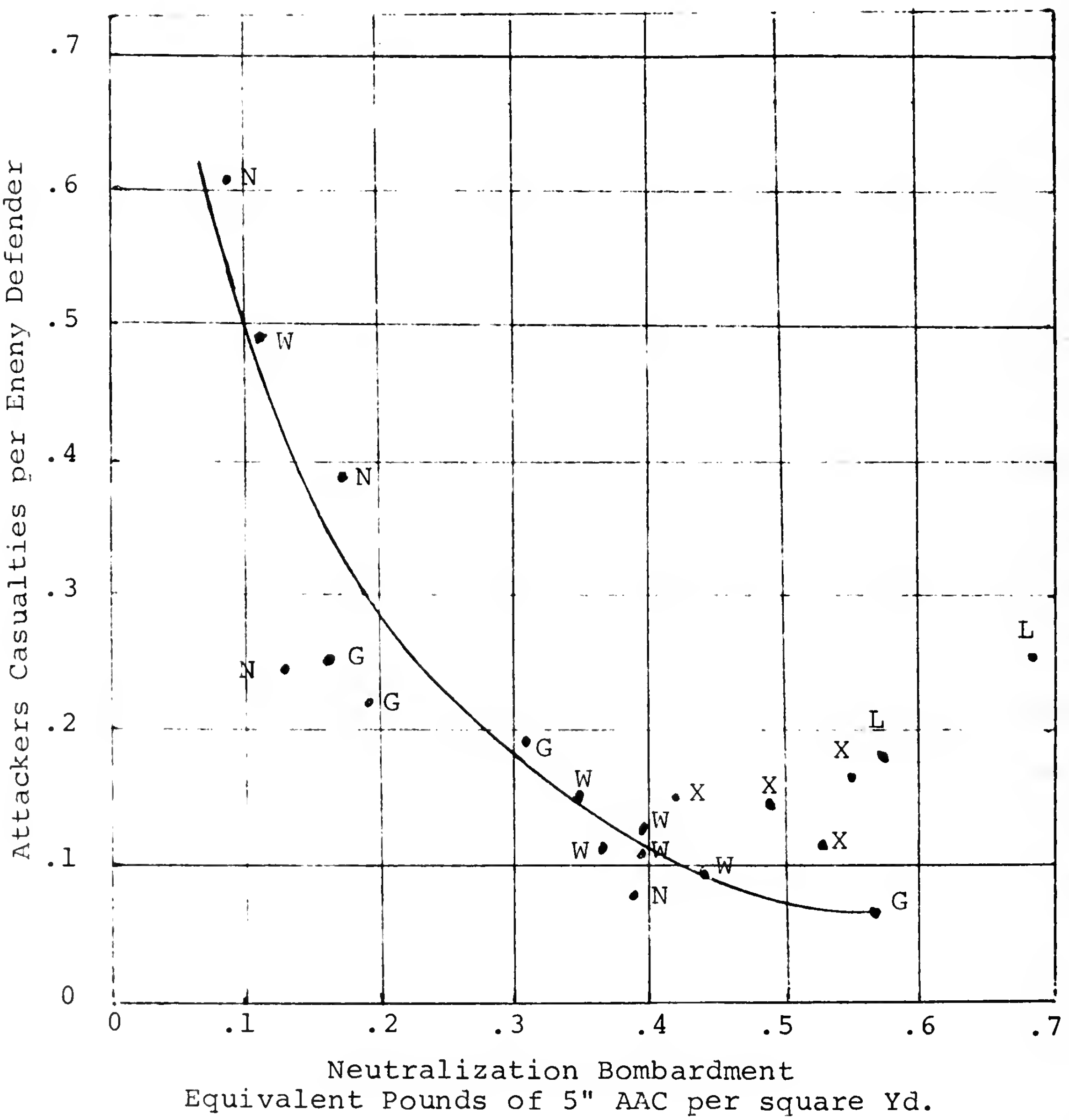

FIGURE 1. EFFECT OF TOTAL AMOUNT OF BOMBARDMENT ON ATTACKERS CASUALTIES

"got well behind the shelling for one reason or another found the enemy recovering and beginning to resist."

Shellard and Welch also showed that the data from these battles could be described by the mathematical relationship $\frac{dN}{dt} = AN + B(D-N)$, where N is the number of effective troops, D is the total number of troops, A is a factor associated with the effect of the bombardment and B is a factor associated with the ability of the defenders to again become effective after dropping out. The data considered indicated that B, the recovery factor, was negligible during the bombardment (Ref. 19).

The historical evidence seems to support the following conclusions: the drop-out phenomenon is a significant cause for diminished fire power in combat and may be the most important reason for depressed volume of fire; the drop-out phenomenon is related to the individuals weapon efficiency; the drop-out phenomenon can be effected by battlefield procedures; and, the drop-out phenomenon is effected by the weapons and volume of fire of the opposing forces. A model of the combat process should, therefore, include consideration of the drop-out phenomenon.

III. VARIABILITY OF THE COMBAT PROCESS

Lanchester's equations describe the variability of the combat process due to attrition. Some of the deterministic extensions of Lanchester's theory have considered other sources of variability. For example, Schaffer considered variability in weapon-efficiency coefficients due to the action of ambushees in seeking cover (Ref. 21). Stochastic models have considered attrition to be the only variability in the combat process.

Examination of the combat process shows that battle tires those engaged in it. Bills notes three types of fatigue which are present in individuals involved in combat. "Reactive fatigue: which results from continuous performing or reaction to stimuli and involves a reduced capacity to perform. Postural or attitudinal fatigue: which results from the prolonged maintenance of such sensori-neuro-muscular postures as attention, alertness, task sets, apprehensiveness, anxiety, etc., and involves a reduced capacity to maintain these postures or attitudes. Motivational fatigue: which is caused by a weakening of the main drive toward the task or of supporting drives, or an increase in strength of competing drives, such as desire for rest, desire for change, escape from danger, longing for home, sexual satisfaction, etc." (Ref. 5).

Total exhaustion in combat was investigated by Beebe and Appel. They analyzed the susceptibility of 1000 men in rifle companies to combat exhaustion in World War II. Significant statistical correlation between exhaustion and both the intensity of the combat on a given day and the total number of combat days in the theater was found. The data supported the conclusion that all men are susceptible to fatigue which renders them incapable of functioning in the combat process (Ref. 3). This suggests that every individual's capability in combat decreases with the intensity of the battle and the total duration of his recent combat experience. Fatigue could be the explanation of the decreasing casualty incidence with time in battle noted by Best.

An experiment performed with three rifle companies in Korea also indicated that psychophysical stress varies with the intensity of combat. Biochemical tests on the body fluids of the men were used to determine the physiological state. Not only were greater changes in body state noted for the more intense battle, the duration of the abnormal state was longer. One company showed pronounced physiological changes for up to three weeks (Ref. 8).

Marshall gives this example of stress causing a deterioration of physical capability in Korea. "Love Company started out . . . at 7:00 in the morning, and from their assembly point it was not more than four or five hundred yards until they started up the ridge. This, incidentally, is a strong company . . . from a very strong regiment, the 27th Wolfhound Regiment. Physically the men were in good shape. The ridge was of course very steep, but counting trail distance and not map distance, they only had approximately 1100 yards to go up to the point where they engaged the Chinese. The fire engagement actually started around 8:30 in the morning. From that time on until 5:00 in the afternoon, when the last Chinese machine gun post in this center was knocked out, Love Company was constantly engaged. As you get into the testimony of what happened to these platoons and squads from 11:30 on, you find the leaders, sergeants, and officers saying that their greatest difficulty - mind you, this is in broad daylight under mortar and intense bullet fire - their greatest difficulty during the hours of crisis in the afternoon is that they cannot keep their men awake. Men who are in the act of firing will fall asleep, even though being asleep increases their danger." (Ref. 5).

It seems reasonable that a unit can also increase its capability to destroy the enemy during the battle. Tactics, for example, are predominately maneuvers to change the geometry associated with the combat process, so that the abilities of the unit are enhanced and the power of the enemy is reduced. During a battle, stimuli are received from the enemy which give information about location, strength, composition, and procedures. By processing this information, a unit can increase its battle capability. Learning during battle can also increase abilities which may account for the high premium placed on "battle hardened" troops.

A combat model which only describes the variability of the combat process due to attrition neglects the other sources of variability. The evidence suggests that these sources of variability may have significant effect on the combat process.

IV. THE STOCHASTIC MODEL

Since the predominant purpose in postulating a model of combat is the predicting of future outcomes of the process, one object of primary interest is the probability that at some time the process is in a given state. With the statement of a criterion of termination, victory probabilities may be found, as well as number of survivors. A state in this model will be the 4-tuple (r,s,b,c), where r is the number of red casualties, s is the number of red dropouts, b is the number of blue casualties, and c is the number of blue dropouts. At some time t_0 the process is in state (r_0,s_0,b_0,c_0). If m_s is the number of red soldiers committed to the battle and n_s is the number of blue soldiers committed to the battle, then at time t_0 there are $m_0 = m_s - r_0 - s_0$ effective red soldiers and $n_0 = n_s - b_0 - c_0$ effective blue soldiers.

Because the capability of the units involved in combat are not constant, the probability of $r - r_0$ red casualties, $s - s_0$ red dropouts, $b - b_0$ blue casualties and $c - c_0$ blue dropouts in a period of length t depends not only upon t, but also on the process time t_0 at which the length t begins. This probability will be designated by

$$v(t_0,t;r-r_0,s-s_0,b-b_0,c-c_0).$$

Thus

$$v(t_0,t;r-r_0,s-s_0,b-b_0,c-c_0)$$

is the conditional probability that during the period (t_0,t_0+t) there occur $r - r_0$ red casualties, $s - s_0$ red dropouts, $b - b_0$ blue casualties, and $c - c_0$ blue dropouts given that at time t_0 there were m_0 red effectives, s_0 red dropouts, n_0 blue effectives, and c_0 blue dropouts.

For simplicity, and as Shellard and Welch found the recovery factor to be negligible, we shall consider that dropping out is not reversible, that is, a dropout may not become an effective while the process continues. Casualties can be generated from the population of dropouts.

Let $W_r(t_0,t) = v(t_0,t;1,0,0,0)$ denote the probability that during (t_0,t_0+t) exactly one effective red unit becomes a casualty and no dropouts or blue casualties occur. Let

$$W_s(t_0,t) = v(t_0,t;0,1,0,0)$$

denote the probability that during (t_0,t_0+t) exactly one effective red unit becomes a dropout and no casualties or blue dropouts occur. Let

$$W_k(t_0,t) = v(t_0,t;1,-1,0,0)$$

denote the probability that during (t_0,t_0+t) exactly one red dropout becomes a casualty and no dropouts or blue casualties occur.

In a similar manner for blue, one may define:

$$W_b(t_0,t) = v(t_0,t;0,0,1,0),$$

$$W_c(t_0,t) = v(t_0,t;0,0,0,1), \text{ and}$$

$$W_j(t_0,t) = v(t_0,t;0,0,1,-1) .$$

The probability that more than one casualty or dropout occurs during a time interval of length (t_0,t_0+t) is

$$U(t_0,t) = 1 - v(t_0,t;0,0,0,0) - v(t_0,t;1,0,0,0)$$
$$- v(t_0,t;0,1,0,0) - v(t_0,t;1,-1,0,0)$$
$$- v(t_0,t;0,0,1,0) - v(t_0,t;0,0,0,1)$$
$$- v(t_0,t;0,0,1,-1).$$

Here the subscript j is associated with the probability that a blue dropout becomes a casualty and the subscript k is associated with the probability that a red dropout becomes a casualty.

The resultant process will be called orderly if as $t \to 0$, for any constant $t_0 \geq 0$, the following relation holds:

$$\frac{U(t_0,t)}{t} \to 0.$$

This assumption is equivalent to stating that the probability of more than one casualty or dropout occurring at the same instant is zero.

Furthermore, it is assumed that, for any $t_0 \geq 0$, there exists

$$\lim_{t\to 0} \frac{W_i(t_0,t)}{t} = C_i(t_0),\ i = r,s,k,b,c,j,$$

the instantaneous value of the parameter (a time rate). Note that the number of effective personnel and the number of dropouts at process time $t_0, (m_0,n_0,s_0,c_0)$, are implied arguments of both the probabilities and the rates.

With these assumptions it is possible to find an expression for the probability that in the interval (t_0,t_0+t) no casualties nor dropouts occur. For any $\Delta t \geq 0$,

$$v(t_0,t+\Delta t;0,0,0,0) = v(t_0,t;0,0,0,0)\ v(t_0+t,\Delta t;0,0,0,0). \quad (1)$$

Let I be the set of indices r,s,k,b,c,j. Then according to the above assumptions, as $\Delta t \to 0$ and when t_0 and t are constants,

$$v(t_0+t,\Delta t;0,0,0,0) = 1 - U(t_0+t,\Delta t) - \sum_I W_i(t_0+t,\Delta t) ,$$

$$= 1 - \sum_I C_i(t_0+t)\Delta t + o(\Delta t) . \tag{2}$$

Substituting (2) into (1) and rearranging terms,

$$v(t_0,t+\Delta t;0,0,0,0) - v(t_0,t;0,0,0,0)$$

$$= -v(t_0,t;0,0,0,0) \quad \sum_I C_i(t_0+t)\Delta t+o(\Delta t) \quad .$$

Dividing both sides by Δt gives in the limit the relation

$$\frac{\partial v(t_0,t;0,0,0,0)}{\partial t} = -v(t_0,t;0,0,0,0) \sum_I C_i(t_0+t) \quad ,$$

(whereby the existence of the right derivative is incidentally shown). Hence.

$$\frac{\partial\{\ln[v(t_0,t;0,0,0,0)]\}}{\partial t} = - \sum_I C_i(t_0+t) \quad ,$$

and as a consequence,

$$\ln[v(t_0,t;0,0,0,0)] - \ln[v(t_0,0;0,0,0,0)]$$

$$= - \int_0^t \sum_I C_i(t_0+u)\,du \quad .$$

Since $\ln[v(t_0,0;0,0,0,0)]$ is zero,

$$v(t_0,t;0,0,0,0) = \exp\{-\int_0^t \sum_I C_i(t_0+u)\,du\}.$$

Consider now the case when any of r,s,k,b,c,j are different from zero. In the same manner as the preceding it follows that as $\Delta t \to 0$ and if t_0 and t are constants,

$$v(t_0,t+\Delta t;r,s,b,c) = v(t_0,t;r,s,b,c)\,v(t_0+t,\Delta t;0,0,0,0)$$
$$+\ v(t_0,t;r-1,s,b,c)\,v(t_0+t,\Delta t;1,0,0,0)$$
$$+\ v(t_0,t;r,s-1,b,c)\,v(t_0+t,\Delta t;0,1,0,0)$$
$$+\ v(t_0,t;r-1,s+1,b,c)\,v(t_0+t,\Delta t;1,-1,0,0)$$
$$+\ v(t_0,t;r,s,b-1,c)\,v(t_0+t,\Delta t;0,0,1,0)$$
$$+\ v(t_0,t;r,s,b,c-1)\,v(t_0+t,\Delta t;0,0,0,1)$$
$$+\ v(t_0,t;r,s,b-1,c+1)\,v(t_0+t,\Delta t;0,0,1,-1). \qquad (3)$$

Now according to the assumptions the second term of each product in the above sum can be written in terms of the rates. Since these rates are functions of the number of effectives for red and blue, m and n respectively and of the number of dropouts, s and c respectively, these arguments will be stated explicitly as $C_i(t_0+t,m,n,s,c)$. As before,

$$v(t_0+t,\Delta t;0,0,0,0) = 1 - \sum_I C_i(t_0+t,m,n,s,c)\Delta t \ + o(\Delta t).$$

The other terms become:

$$v(t_0+t,\Delta t;1,0,0,0) = C_r(t_0+t,m+1,n,s,c)\Delta t + o(\Delta t),$$
$$v(t_0+t,\Delta t;0,1,0,0) = C_s(t_0+t,m+1,n,s-1,c)\Delta t + o(\Delta t),$$
$$v(t_0+t,\Delta t;1,-1,0,0) = C_k(t_0+t,m,n,s+1,c)\Delta t + o(\Delta t),$$
$$v(t_0+t,\Delta t;0,0,1,0) = C_b(t_0+t,m,n+1,s,c)\Delta t + o(\Delta t),$$
$$v(t_0+t,\Delta t;0,0,0,1) = C_c(t_0+t,m,n+1,s,c-1)\Delta t + o(\Delta t), \text{ and}$$
$$v(t_0+t,\Delta t;0,0,1,-1) = C_j(t_0+t,m,n,s,c+1)\Delta t + o(\Delta t).$$

Substituting into (3) leads to the result:

$$\frac{\partial v(t_0,t;r,s,b,c)}{\partial t} = -v(t_0,t;r,s,b,c) \sum_I C_i(t_0+t,m,n,s,c)$$
$$+ v(t_0,t;r-1,s,b,c)C_r(t_0+t,m+1,n,s,c)$$
$$+ v(t_0,t;r,s-1,b,c)C_s(t_0+t,m+1,n,s-1,c)$$
$$+ v(t_0,t;r-1,s+1,b,c)C_k(t_0+t,m,n,s+1,c)$$
$$+ v(t_o,t;r,s,b-1,c)C_b(t_0+t,m,n+1,s,c)$$
$$+ v(t_0,t;r,s,b,c-1)C_c(t_0+t,m,n+1,s,c-1)$$
$$+ v(t_0,t;r,s,b-1,c+1)C_j(t_0+t,m,n,s,c+1).$$

Given that a change of state occurs in the interval $(t_0+t,t_0+t+\Delta t)$, the conditional probability that the change of state (transition) is of type i is

$$q_i(m,n,s,c) = \frac{C_i(t_0+t,m,n,s,c)}{\sum_I C_i(t_0+t,m,n,s,c)} \quad , \ i=r,s,k,b,c,j. \qquad (4)$$

These probabilities as well as the probability of no transitions in the interval (t_0,t_0+t) suggest that Monte Carlo techniques could be applied to approximate a solution for the desired probabilities. Since no assumptions about the size of the forces participating in the combat process were necessary, an instantaneous rate can be associated with each individual. With the assumption that the individual rates are stochastically independent, then the instantaneous rate for the force is the sum of these individual rates. It seems reasonable to consider that the rates for a population of soldiers had some probability distribution so that the instantaneous rates are random variables. In a simulation, it would be appropriate to sample from that population. The complexity of the combat process implies that determination of the rates even for an individual will be an arduous task which will require research during the combat process rather than investigation of the outcomes of historical battles.

With the restrictive assumption that the instantaneous rates of the process do not depend on time, the probabilities that the process is in a given state at time t can be readily found. The transition probabilities may be computed from (4), with $C_i(t_0+t,m,n,s,c) = C_i(m,n,s,c)$. It is immediately apparent that given the present state of the process, the future states do not depend on the past which means that the process has the Markov property.

The probability that no transitions occur in time t becomes

$$\exp\{-t \sum_I C_i(m,n,s,c)\}.$$

Consider the random variable T whose value is the length of time during which the process remains in a given state. Then,

$$\begin{aligned} F(x) &= \Pr\{T \le x\} \\ &= 1 - \mathrm{Br}\{\text{no transitions occur during time interval } x\} \\ &= 1 - \exp\{-x \sum_I C_i(m,n,s,c)\}, \quad x \ge 0. \end{aligned}$$

The waiting time is recognized as an exponential variate and the process is a stable, continuous time parameter Markov process. This type of process has been explored in the literature on stochastic processes. (Ref. 2,12,13).

The further restrictive assumption that the instantaneous rates are almost constant, that is,

$$C_i(m,n,s,c) = \begin{cases} C_i & m,n > 0 \\ 0 & m,n = 0 \end{cases} \qquad i=r,s,b,c, \text{ and}$$

$$C_i(m,n,s,c) = \begin{cases} C_i & m,n,s,c > 0 \\ 0 & m,n,s,c = 0 \end{cases} \qquad i=k,j$$

results in a stochastic process with expected values which approximate Lanchester's Linear Law.

Consider the embedded Markov chain in this stable, continuous time parameter Markov process with parameter h, (h=0,1,2,...,the times at which transitions occur). The discrete version of equation (3) can be written

$$\begin{aligned} Pr(h,r,s,b,c) = {} & q_r Pr(h-1,r-1,s,b,c) + q_s Pr(h-1,r,s-1,b,c) \\ & + q_k Pr(h-1,r-1,s+1,b,c) + q_b Pr(h-1,r,s,b-1,c) \\ & + q_c Pr(h-1,r,s,b,c-1) + q_j Pr(h-1,r,s,b-1,c+1). \end{aligned}$$

The initial state is known. Furthermore, the process stops when either antagonist's effective forces have been reduced to zero by casualties and dropouts.

These relationships form a set of difference equations with solution,

$$Pr(h,r,s,b,c) = \sum_{k=0}^{J} \frac{h!\, q_r^{r-k}\, q_k^{k}\, q_s^{s+k}\, q_b^{b-j+k}\, q_j^{j-k}\, q_c^{c+j-k}}{(r-k)!k!(s+k)!(b-j+k)!(j-k)!(c+j-k)!},$$

where

$$J = h - (r+s+b+c), \text{ for } r-k, k, s+k, b-j+k, j-k$$

and

$$c+j-k \geq 0.$$

From this solution we find that the expected number of effectives which become casualties (r-k) is $q_r h$ and the expected number of new dropouts in these h transition is $q_s h$. The expected number of red effectives at the hth transition is, therefore, $m_h = m_s - r_0 - s_0 - q_r h - q_s h$ and the expected number of red effectives at the h'th transition is

$$m_{h'} = m_s - r_0 - s_0 - q_r h' - q_s h'.$$

Subtracting and dividing by $\Delta h = h-h'$, we have

$$\frac{\Delta m_h}{\Delta h} = -(q_r+q_s).$$

If the number of effectives are large enough so that the change m_h is small compared with the number of effectives then

$$\frac{\Delta m_h}{\Delta h}$$

is approximately dm/dt. In a similar manner $dn/dt = -(q_b+q_c)$. These equations can be solved to yield

$$m = m_0 - (q_r+q_s)t \text{ and } n = n_0 - (q_b+q_c)t.$$

Now define efficiency (exchange ratio) as

$$E = (q_r+q_s)/(q_b+q_c).$$

Since

$$q_r + q_s + q_k + q_b + q_c + q_j = 1, \ (q_r + q_s) = E(1-q_k-q_j)/(E+1),$$

and

$$(q_b+q_c) = (1-q_k-q_j)/(1+E).$$

Substituting and dividing,

$$dm/dn = E.$$

Integration gives the result $(m-m_0) = E(n-n_0)$, a well known form known as Lanchester's Linear Law.

Thus the model postulated reduces to a stochastic model with expected value which approximates Lanchester's linear equations if very restrictive assumptions are made. These assumptions do not appear to be justified except for battles of very short duration.

IV. CONCLUSIONS

It has been shown that the dropout phenomenon can be and should be considered by a model of combat. For the simple Lanchester model of combat, the dropout phenomenon effects the exchange ratio. As there is reason to believe that the dropout phenomenon is not constant from battle to battle, exchange ratios computed from historical battles should show variability. Willard's conclusion from analyzing 1500 historical land battles that the exchange ratio has controlled the outcome of battle (Ref. 25) indicates that to predict the outcome of the combat process, the exchange ratio must be predicted. This implies that the dropout phenomenon must first be predicted.

An immediate result from the stochastic model with dropouts is combat does not proceed to the annihilation of either side. This result is also evident in historical data (although not necessarily for the same reasons). In only two per cent of the battles considered by Willard did the critical casualty ratio (maximum $(n_0-n_f)/n_0$, $(m_0-m_f)/m_0$, where the subscript f denotes the number at the end of the battle) exceed one half. The annihilation of either side was used as a criterion for predicting the victor of an engagement by Brown (Ref. 7) and Morse and Kimball (Ref. 18).

Since the parameters of combat appear to be variable during the process, research during the process is necessary so that a method by which future outcomes of the process may be predicted can be developed. There is much to discover about the debilitating effects of combat on men which seems to be a major source of variability. Such research can only be conducted during time of war.

Knowing only that the dropout is present in significant numbers on the battlefield does not give complete knowledge about the drop-out phenomenon, but suggests that behavior in combat should be investigated thoroughly by trained observers on the battlefield who have no other function. Understanding the drop-out phenomenon should lead not only to better predictive models, but also to more effective soldiers, procedures and tactics.

The work in this thesis represents a beginning in the development of a stochastic model with time variable parameters. It is hoped that the model postulated will help to motivate work in this area.

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13. ABSTRACT

Stochastic models of combat which have been proposed consider that the personnel on the battlefield participate fully in the battle. Observation of behavior in combat indicates that a significant portion of the forces involved do not fire their weapons. These personnel are termed drop-outs and the drop-out phenomenon is discussed. A probabilistic model of combat which considers both casualty generation and the drop-out phenomenon is proposed. This combat model has time variable rates associated with the attrition of the opposing sides. With the relaxation of this assumption, the stochastic combat model is shown to have an expected value which approximates Lanchester's Linear Law.

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